Dynamic Systems Sea-Level Rise Model: Interpretation of the Forcing Function

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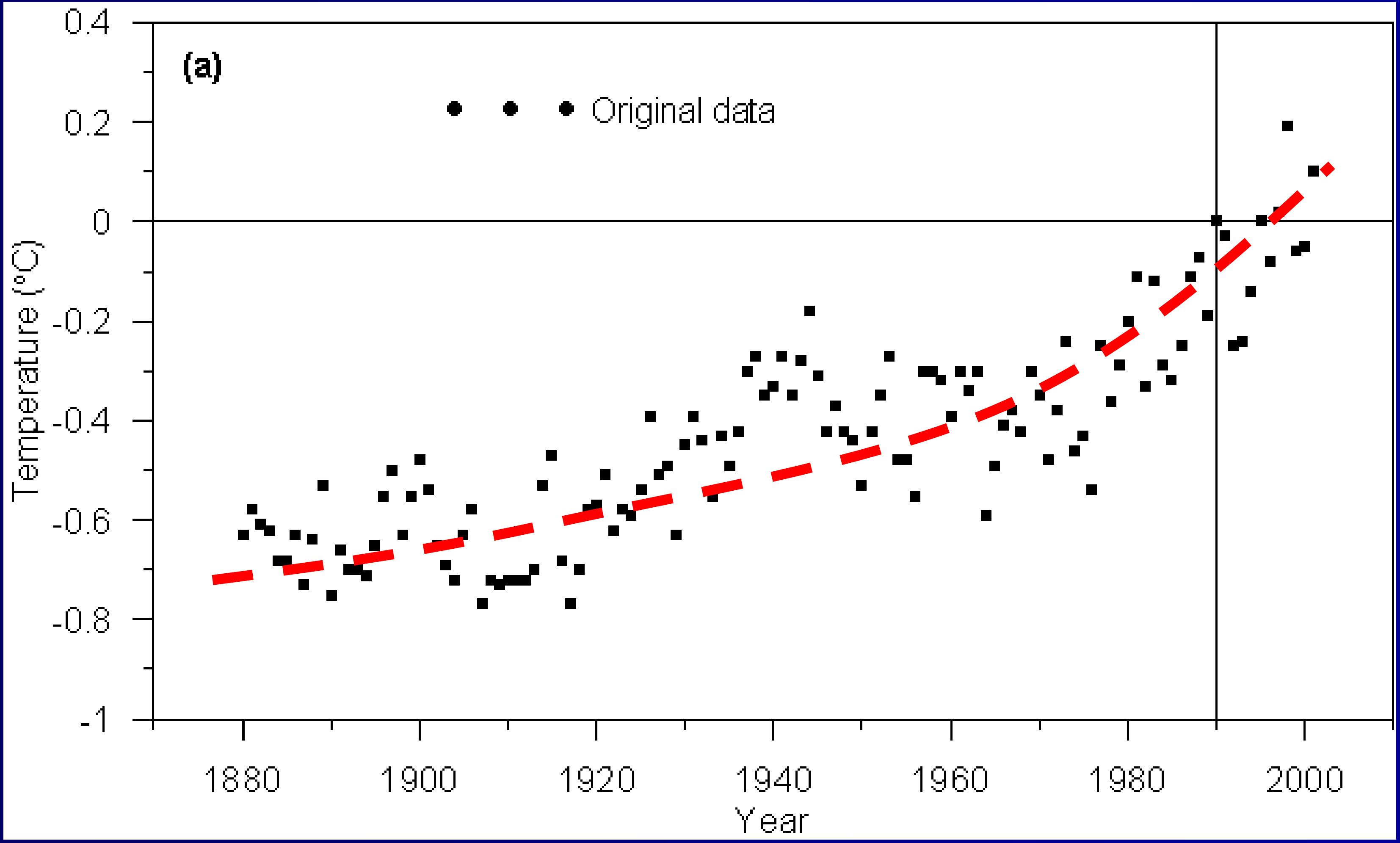
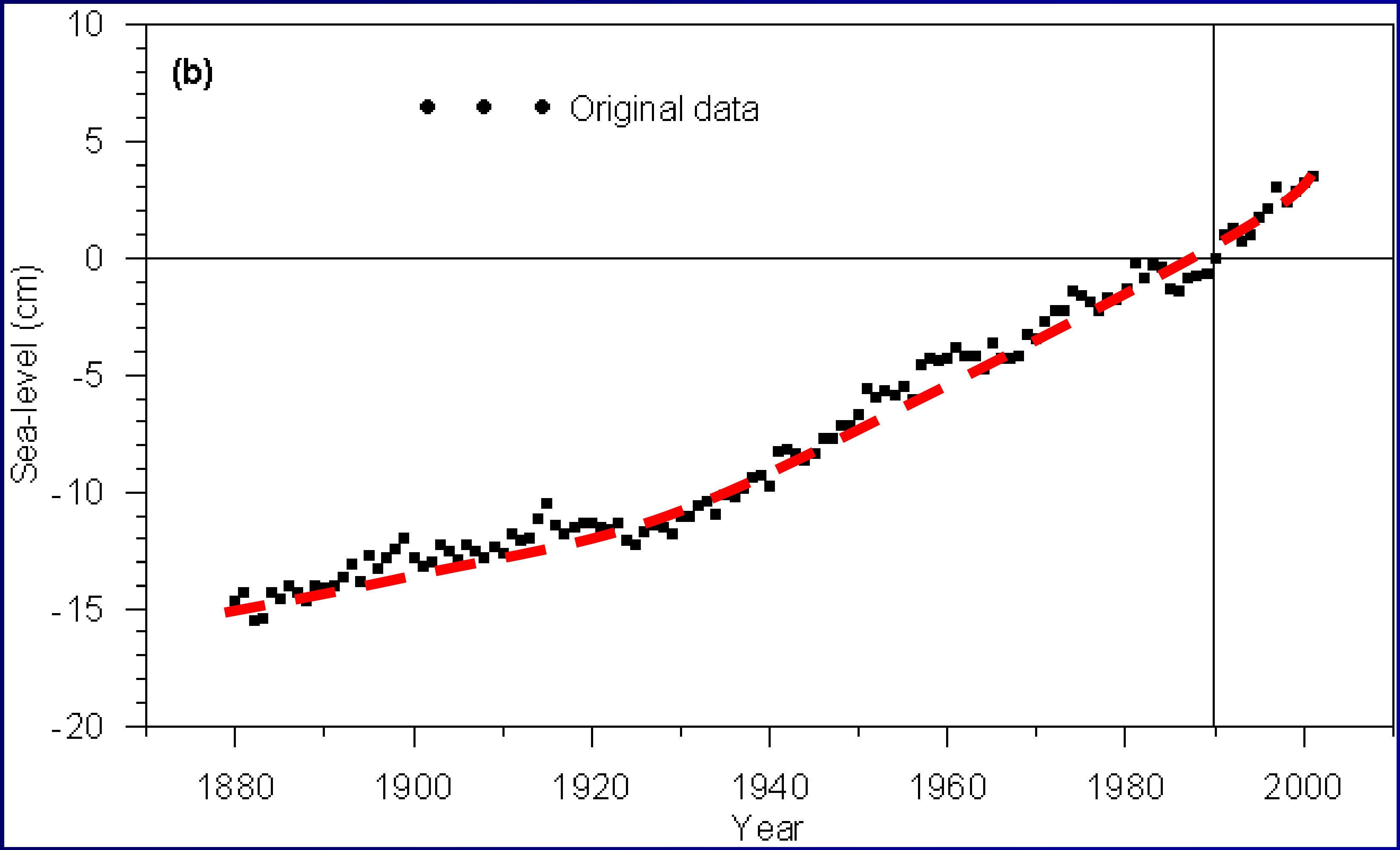
Climate Change / Sea-level Rise DATA on:

Temperature
Sea-Level Rise
Radiative forces (CO_2 Emissions)

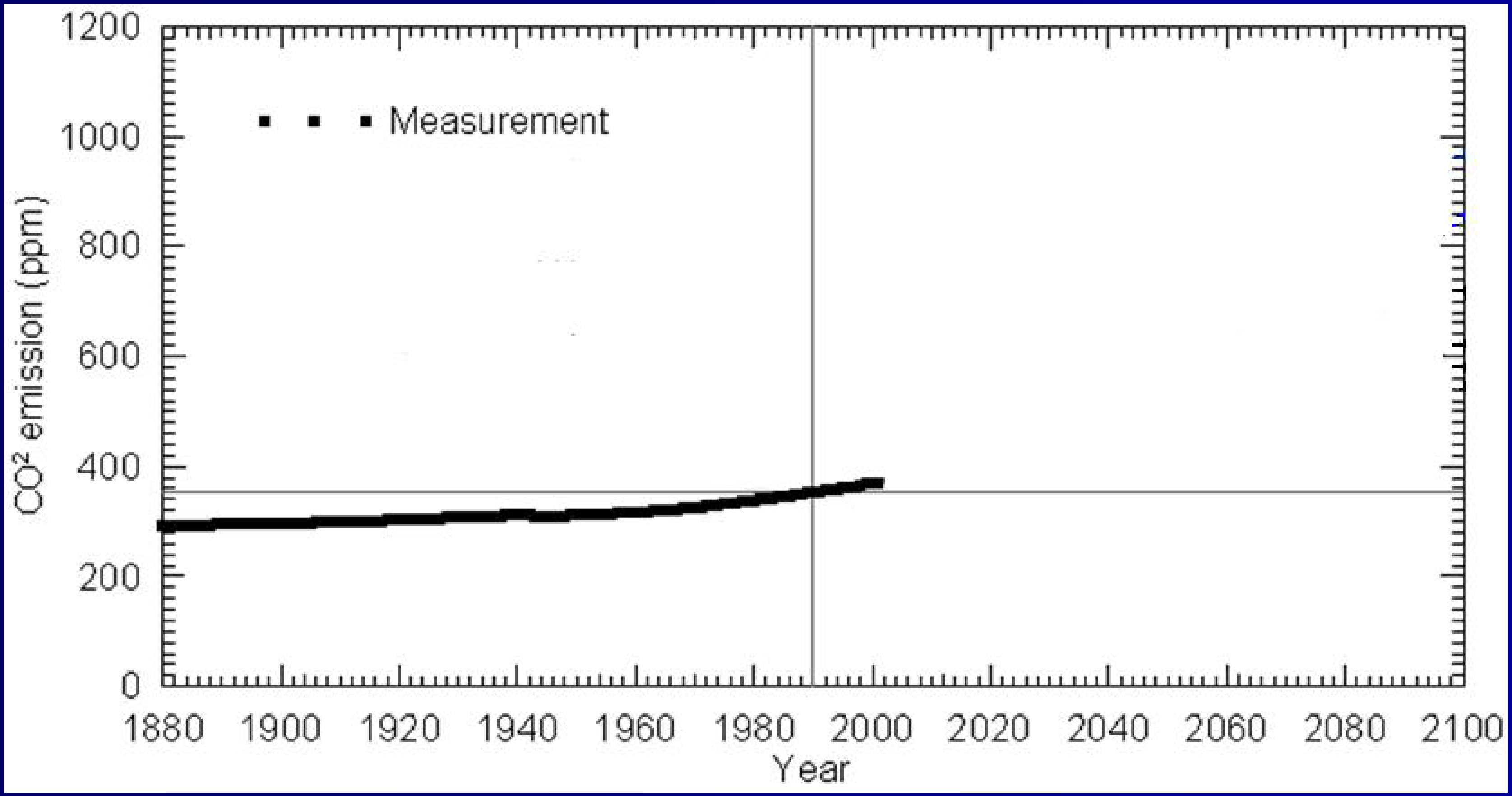
Global Temperature Change:



Global Sea-Level Rise:



Data on CO_2 Emissions:

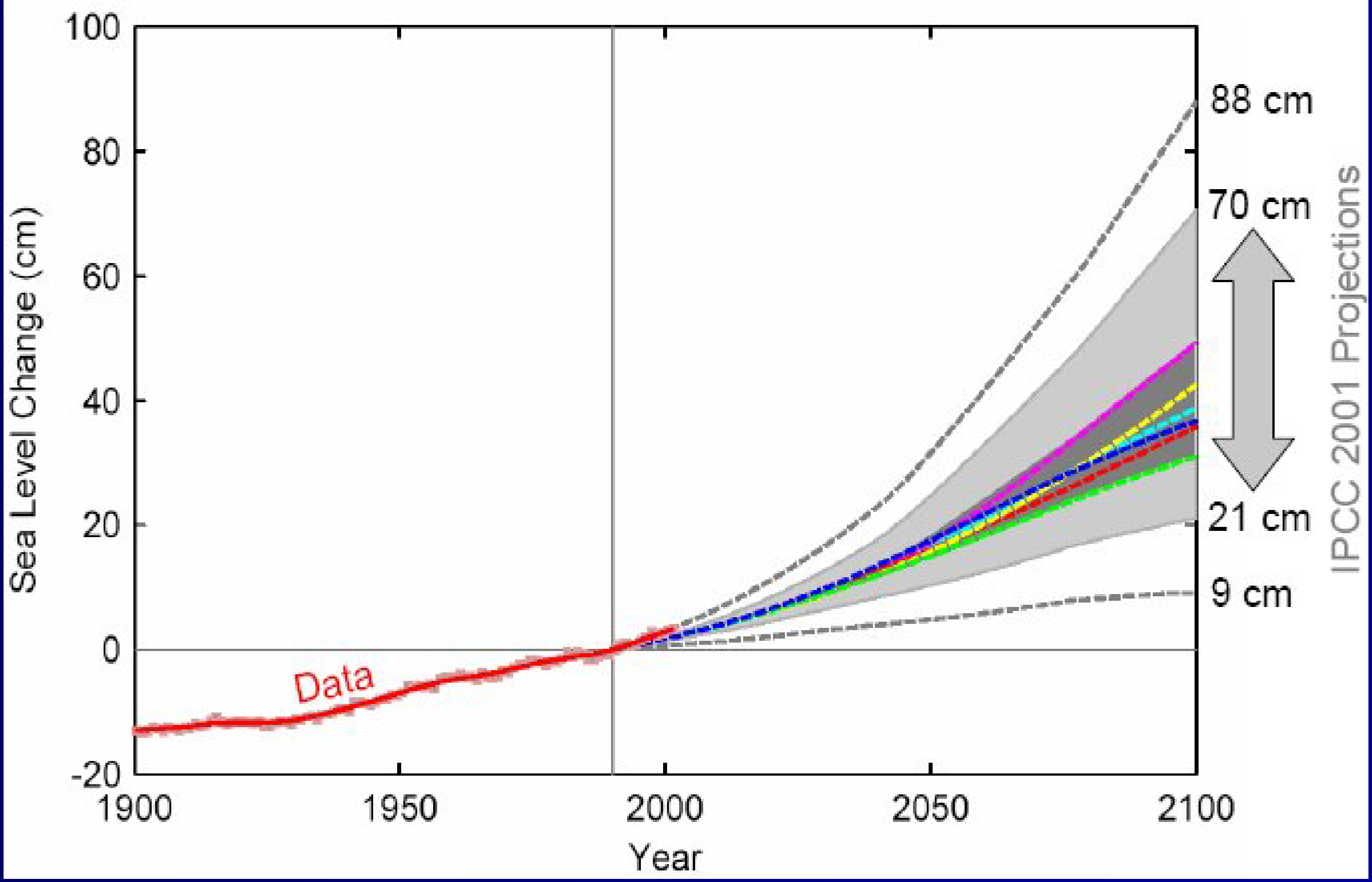


The IPCC Study:

The Summary for Policy Makers (SPM) released recently provide the following table of sea level rise projections (IPCC4th Framework Report, 2007):

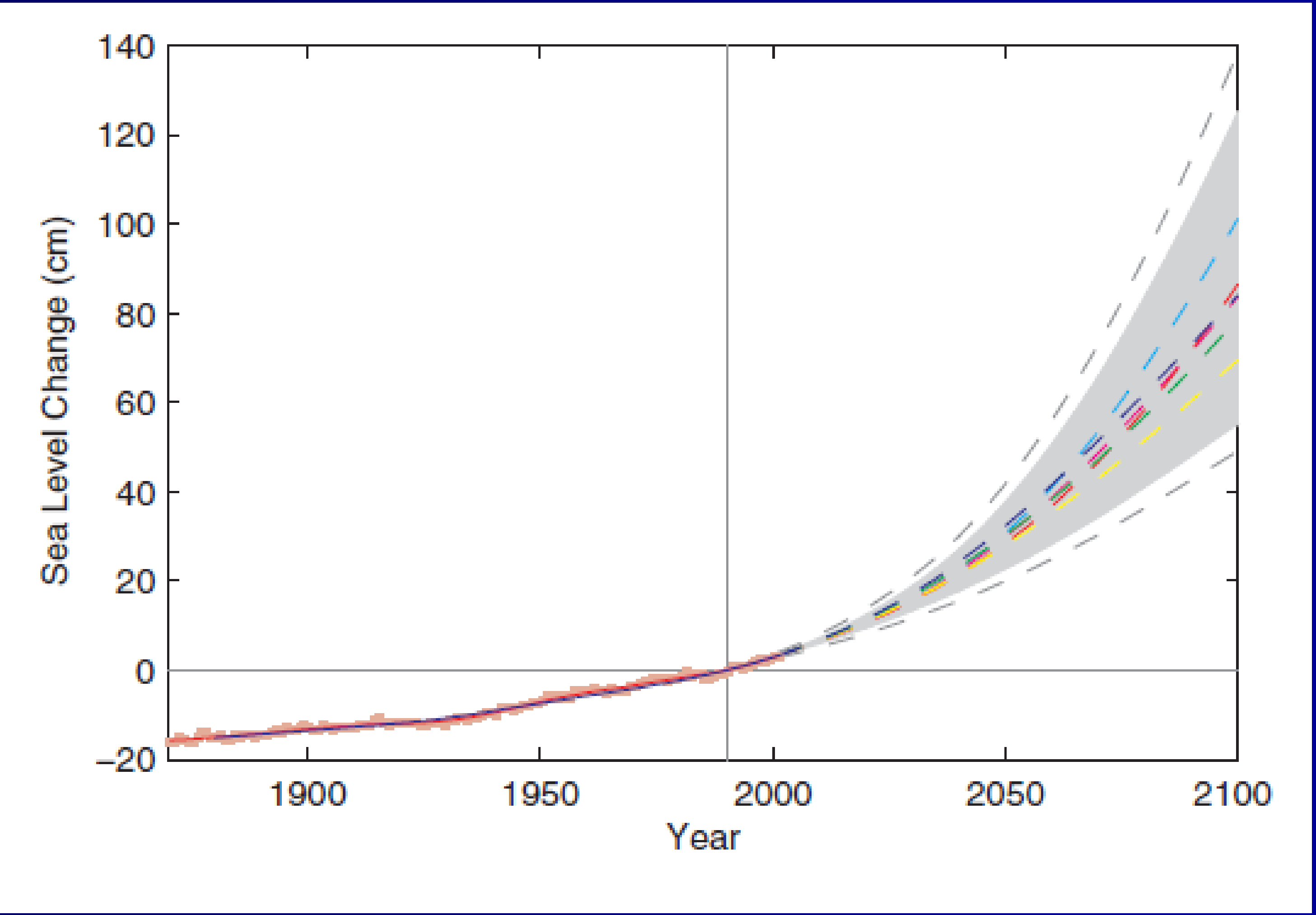
Case	Sea Level Rise (m at 2090-2099 relative to 1980-1999) Model-based range excluding future rapid dynamical changes in ice flow
B1 scenario	0.18 – 0.38
A1T scenario	0.20 – 0.45
B2 scenario	0.20 – 0.43
A1B scenario	0.21 – 0.48
A2 scenario	0.23 – 0.51
A1FI scenario	0.26 – 0.59

IPCC estimates:



Semi-Empirical Models:

Rahmstorf's Study (Science, Vol. 315 pp.19, 2007) and others:



Semi-Empirical Models:

Rahmstorf's Study (Science, Vol. 315 pp.19, 2007) and others.

Sea-level Rise Interval Predicted above 1990 level:

$$\{0.5 - 1.4m\}$$

IPCC Interval Predicted above 1990 level:

$$\{0.09 - 0.88m\}$$

Major limitations of the previous empirical models

- No feedback of SLR on temperature change
- Zero-dimensional, thus does not capture spatial variations in SLR
- Impact of external forcing is not considered

A Dynamic Systems Model

Expected Relationship:

- Earlier studies showed that the relationship between T & H is linear.

Our Hypothesis: (Dynamic Systems Model):

- Temperature: $T = f_1\left(T, H, U, c_1\right)$
- Sea-Level: $H = f_2\left(T, H, U, c_2\right)$

Initial Model Proposed:
(Simplified)

$$\frac{dT(t)}{dt} = a_{11}T(t) + a_{12}H(t) + c_1$$

$$\frac{dH(t)}{dt} = a_{21}T(t) + a_{22}H(t) + c_2$$

Initial Model Proposed:
(Simplified)

$$\frac{d\mathbf{X}(t)}{dt} = \left(\frac{dT(t)}{dt}, \frac{dH(t)}{dt} \right)^{\tau}$$

$$\frac{d\mathbf{X}(t)}{dt} = \mathbf{A}\mathbf{X}(t) + \mathbf{C}$$

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad \mathbf{C} = \begin{Bmatrix} c_1 \\ c_2 \end{Bmatrix}$$

Initial Model Proposed:
(Simplified)

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A Dynamic System Model to Predict Global Sea-Level Rise and Temperature Change

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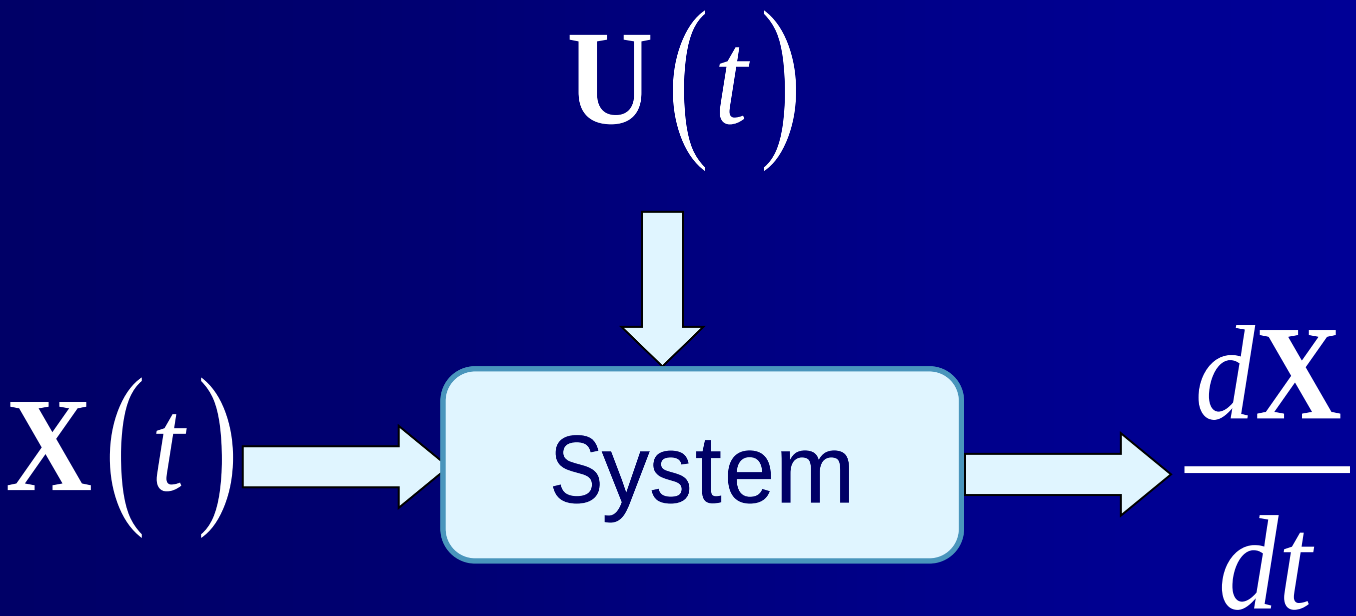
$\mathbf{U}(t)$

$\mathbf{X}(t)$

$\frac{d\mathbf{X}}{dt}$

Dynamic Systems Model:



Proposed Model:

$$\frac{dT(t)}{dt} = a_{11}T(t) + a_{12}H(t) + \sum_i a_{13i}U_i\left(t\right) + c_1$$

$$\frac{dH(t)}{dt} = a_{21}T(t) + a_{22}H(t) + \sum_i a_{23i}U_i\left(t\right) + c_2$$

The Proposed model is:

- more flexible;
- may answer more questions;
- may provide control analysis perspective; and,
- hopefully will be more useful.
- potential drawback may require more data.

Proposed Model:

$$\frac{d\mathbf{X}(t)}{dt} = \mathbf{AX}(t) + \mathbf{BU}(t) + \mathbf{C} + \mathbf{w}(t)$$

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1m} \\ b_{21} & b_{22} & \cdots & b_{2m} \end{bmatrix}$$

$$\mathbf{C} = \begin{Bmatrix} c_1 \\ c_2 \end{Bmatrix}$$

Discrete form of the Proposed Model:

$$\mathbf{X}(k+1)=\mathbf{\Phi X}(k)+\mathbf{\Gamma U}(k)+\mathbf{\Omega}\ \mathbf{w}(k)+\quad+\quad{}_{\mathbf{1}}(k)$$

$$\mathbf{\Phi}=\mathbf{I}+\mathbf{A}\Delta t\ ;\qquad \mathbf{\Gamma}=\mathbf{B}\Delta t\ ;\qquad \mathbf{\Omega}=\mathbf{C}\Delta t$$

LSM Model to determine a_{ij} , b_{ij} & c_i :

$$F^* = \underset{\varphi_i}{\text{minimize}} \left\{ \left(\mathbf{Y}_i - \mathbf{\Lambda}\boldsymbol{\varphi}_i \right)^T \left(\mathbf{Y}_i - \mathbf{\Lambda}\boldsymbol{\varphi}_i \right) \right\}$$

Confidence Interval:

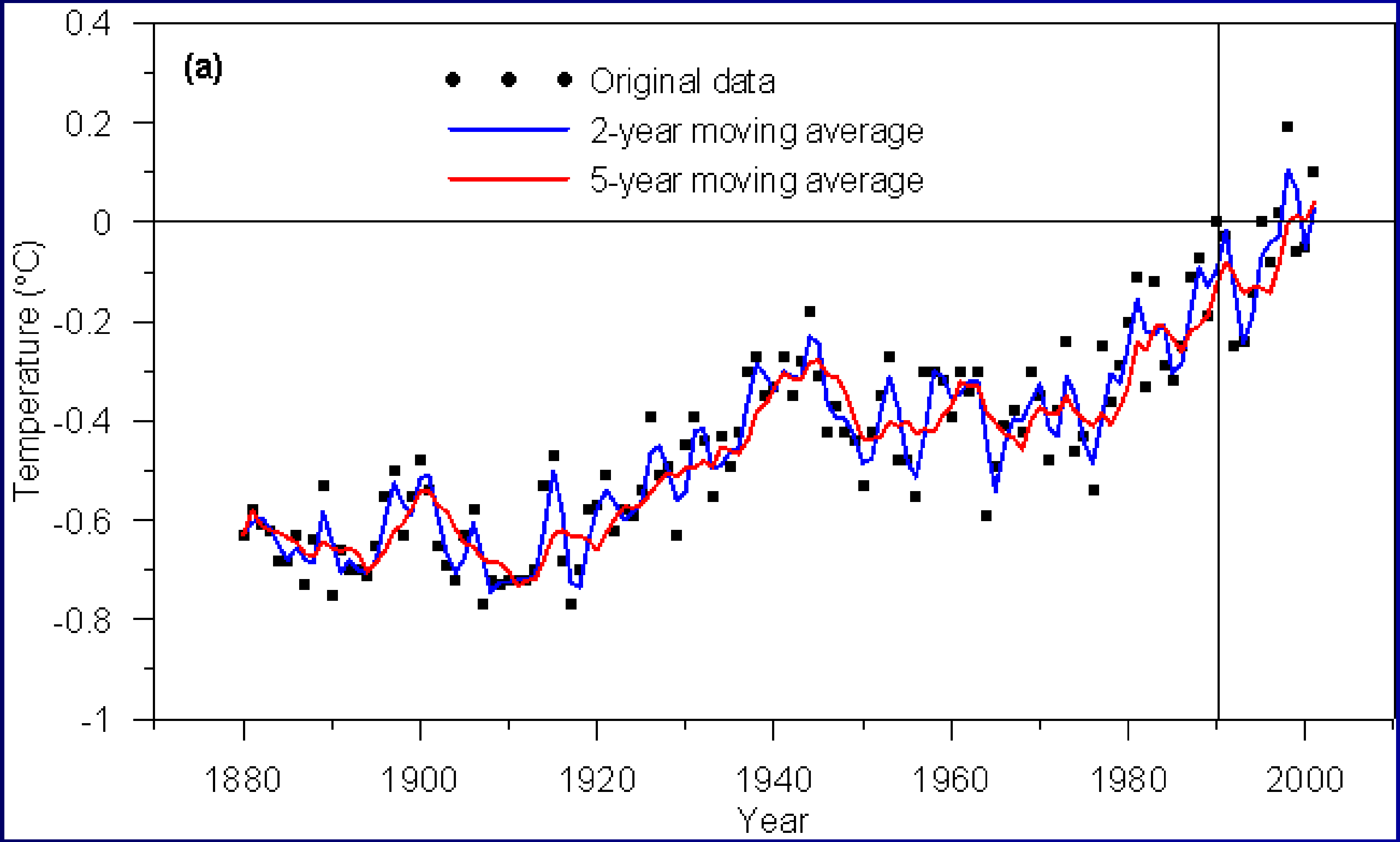
$$\left.\begin{aligned}
\hat{T}_{CI}(k) &= \hat{T}(k) \pm t_{\alpha/2,n-4}\sqrt{\hat{\sigma}_T^2\left(1+\hat{Z}(k)^\tau\left(\mathbf{\Lambda}^\tau\mathbf{\Lambda}\right)^{-1}\hat{Z}(k)\right)} \\
\hat{H}_{CI}(k) &= \hat{H}(k) \pm t_{\alpha/2,n-4}\sqrt{\hat{\sigma}_H^2\left(1+\hat{Z}(k)^\tau\left(\mathbf{\Lambda}^\tau\mathbf{\Lambda}\right)^{-1}\hat{Z}(k)\right)}
\end{aligned}\right\}$$

(Wadsworth, 1998)

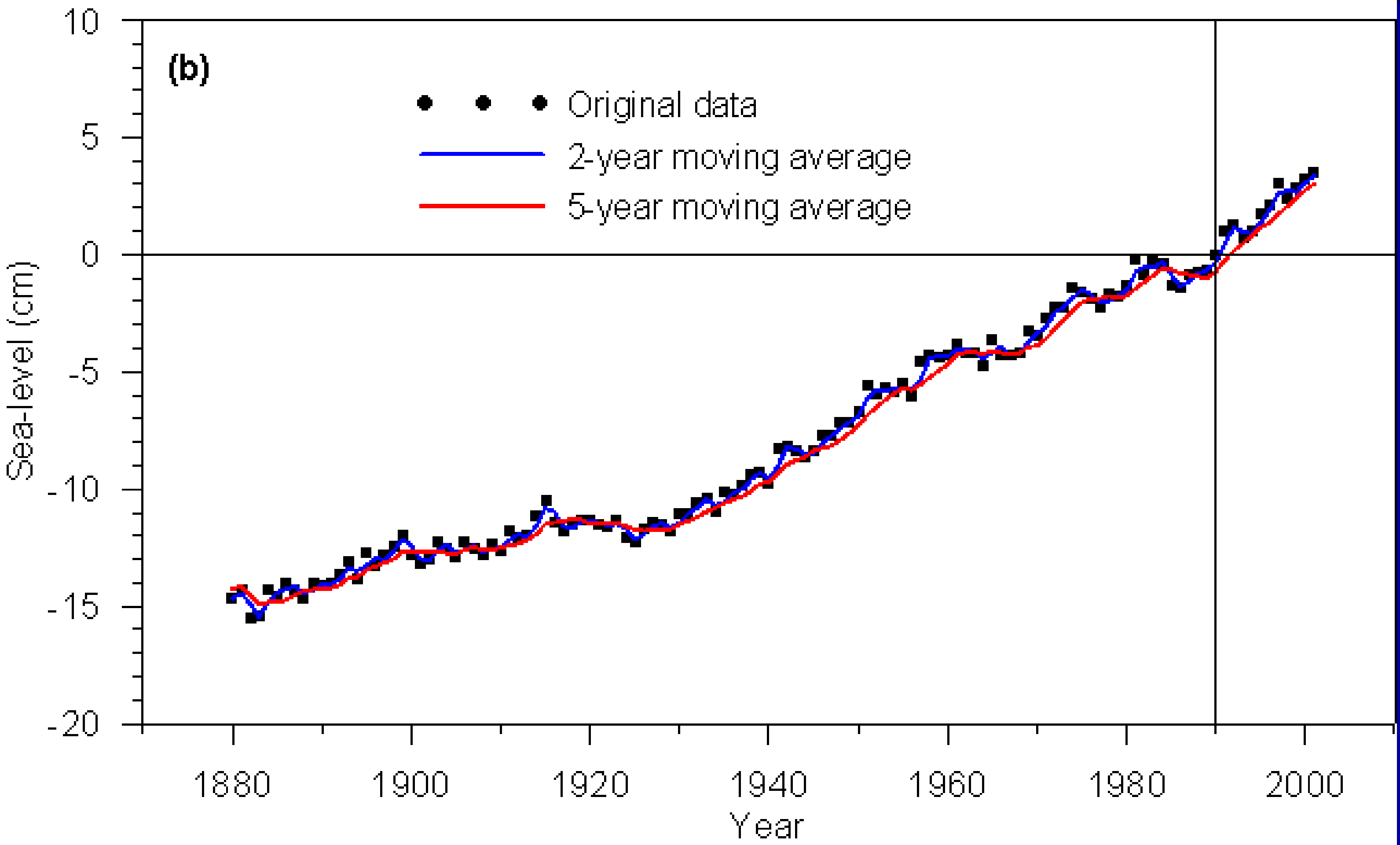
Application:

- 2-year moving average outcome is used for both state variables.

Temperature Data:



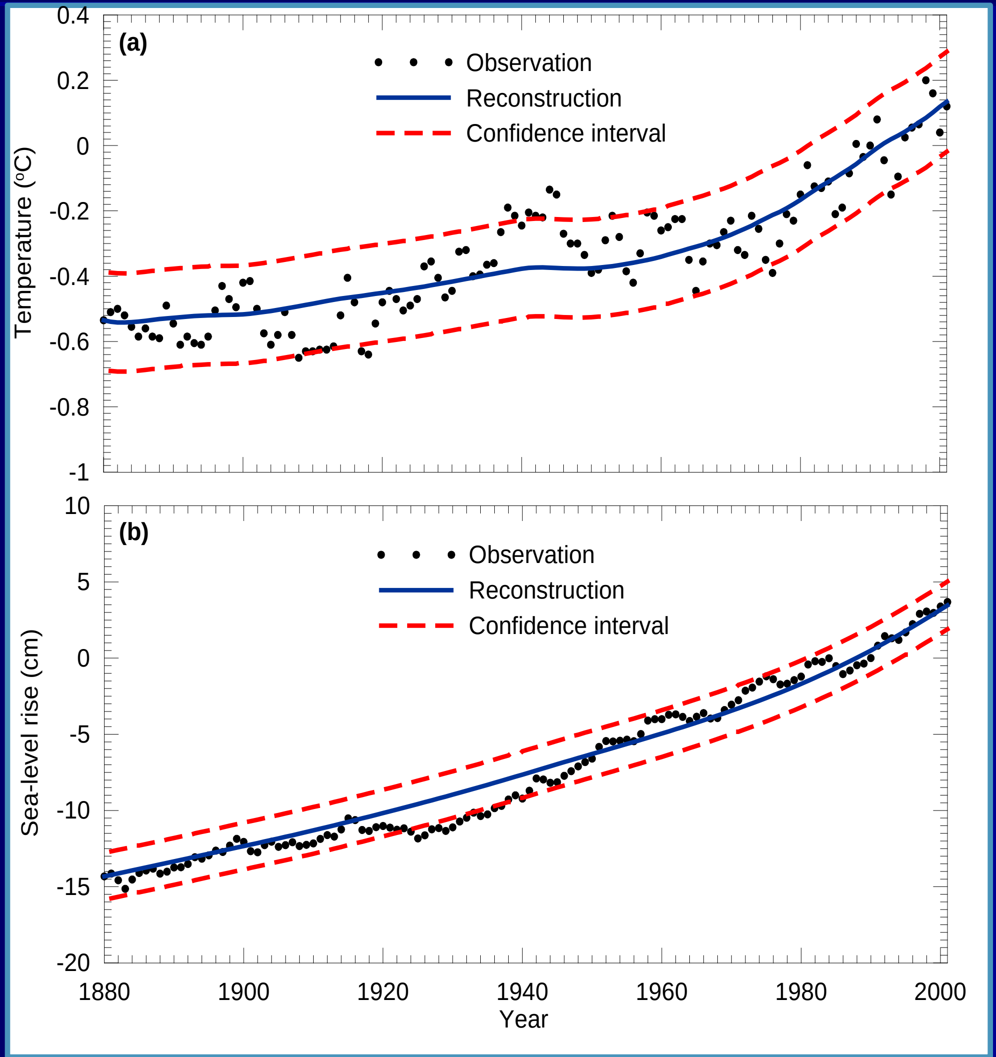
Sea-Level Data:



Resulting Matrix Coefficients:

System	System matrix	Control matrix	Constant vector
Discrete	$\begin{bmatrix} 0.75042 & -0.00053 \\ 0.41015 & 0.99568 \end{bmatrix}$	$\begin{bmatrix} 0.00245 \\ 0 \end{bmatrix}$	$\begin{Bmatrix} -0.85744 \\ 0.25863 \end{Bmatrix}$
Continuous	$\begin{bmatrix} -0.24958 & -0.00053 \\ 0.41015 & -0.00432 \end{bmatrix}$	$\begin{bmatrix} 0.00245 \\ 0 \end{bmatrix}$	$\begin{Bmatrix} -0.85744 \\ 0.25863 \end{Bmatrix}$

Global CO_2 Impact: Calibration

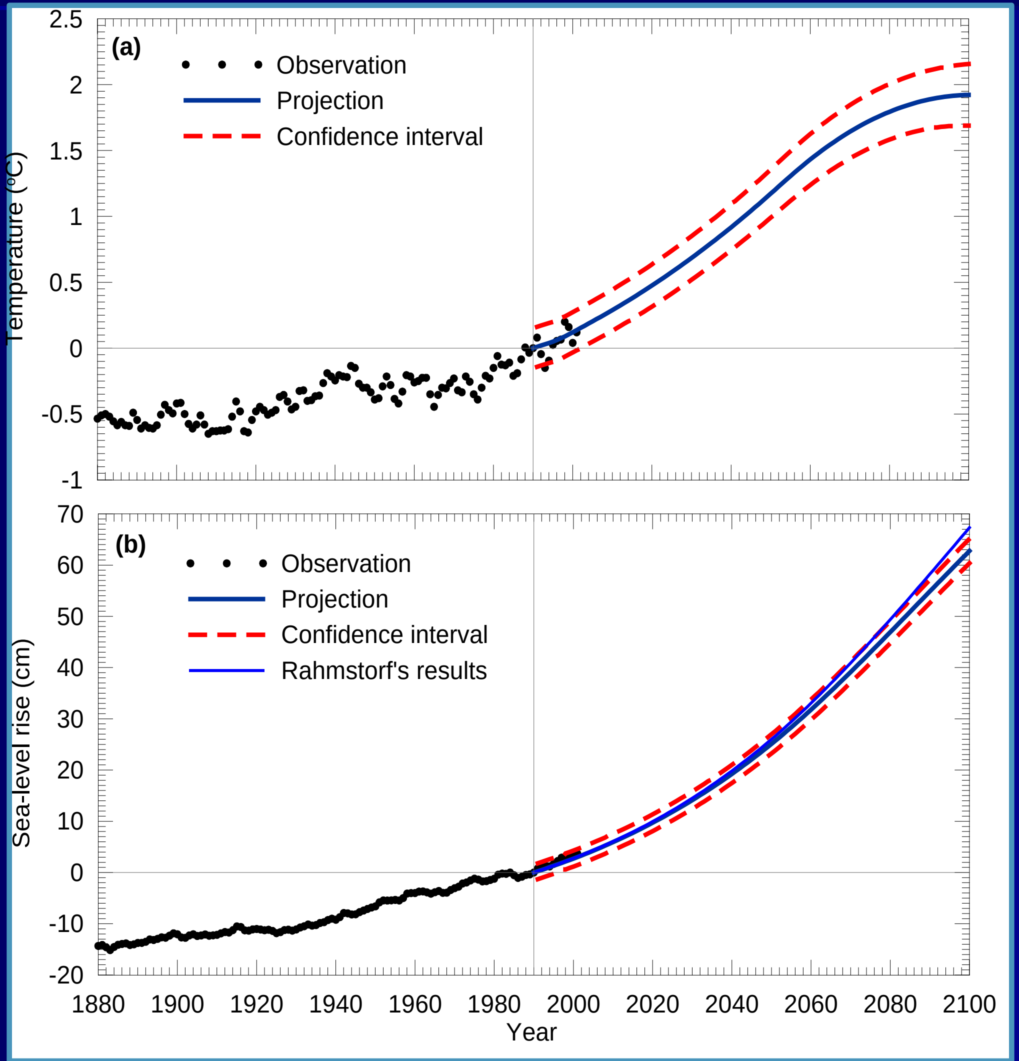


$R^2 = 0.80$

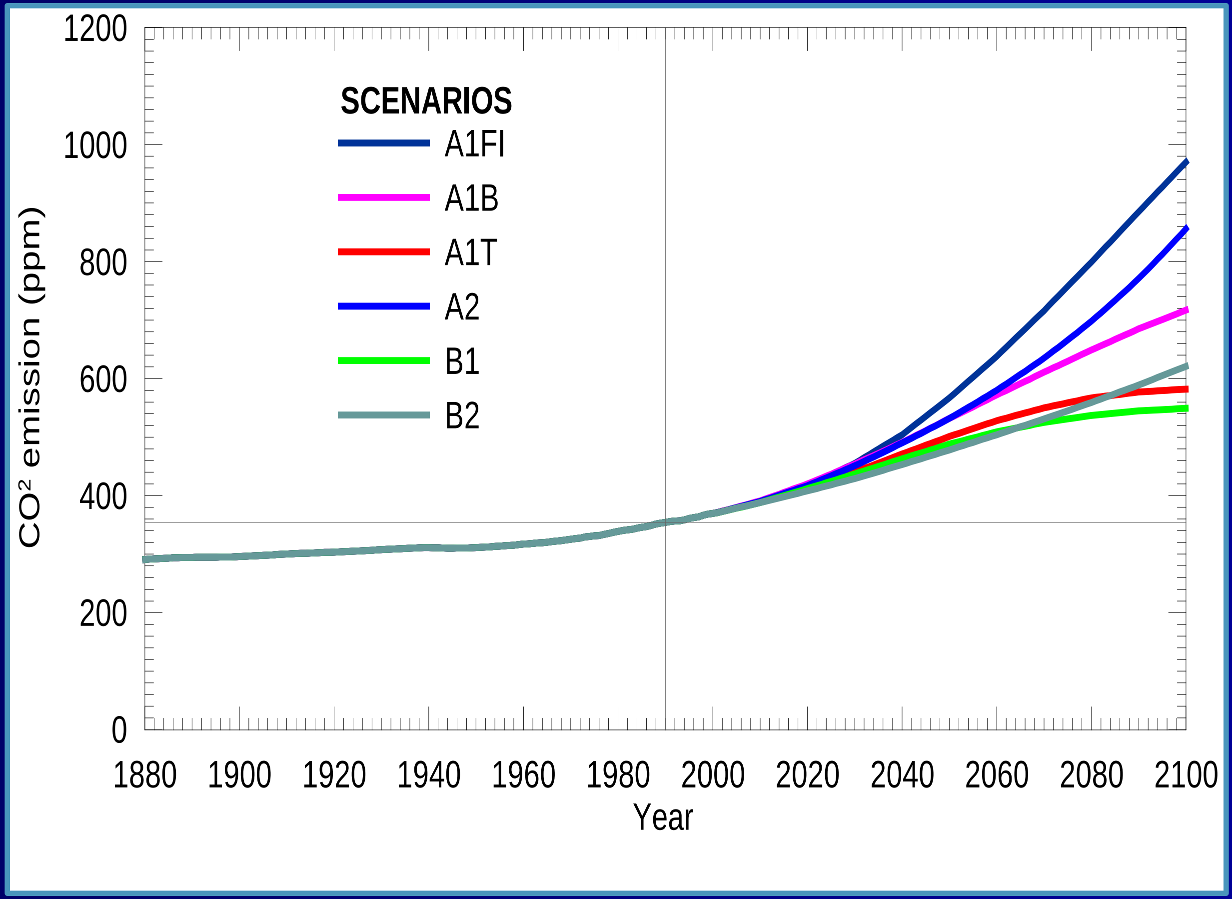
$R^2 = 0.97$

Global CO_2 Impact: Prediction for "2 ºC" Scenario

(Scenario is designed to limit global warming in 2100 2ºC above the temperature in 2000)
(Hansen et al., 2000)



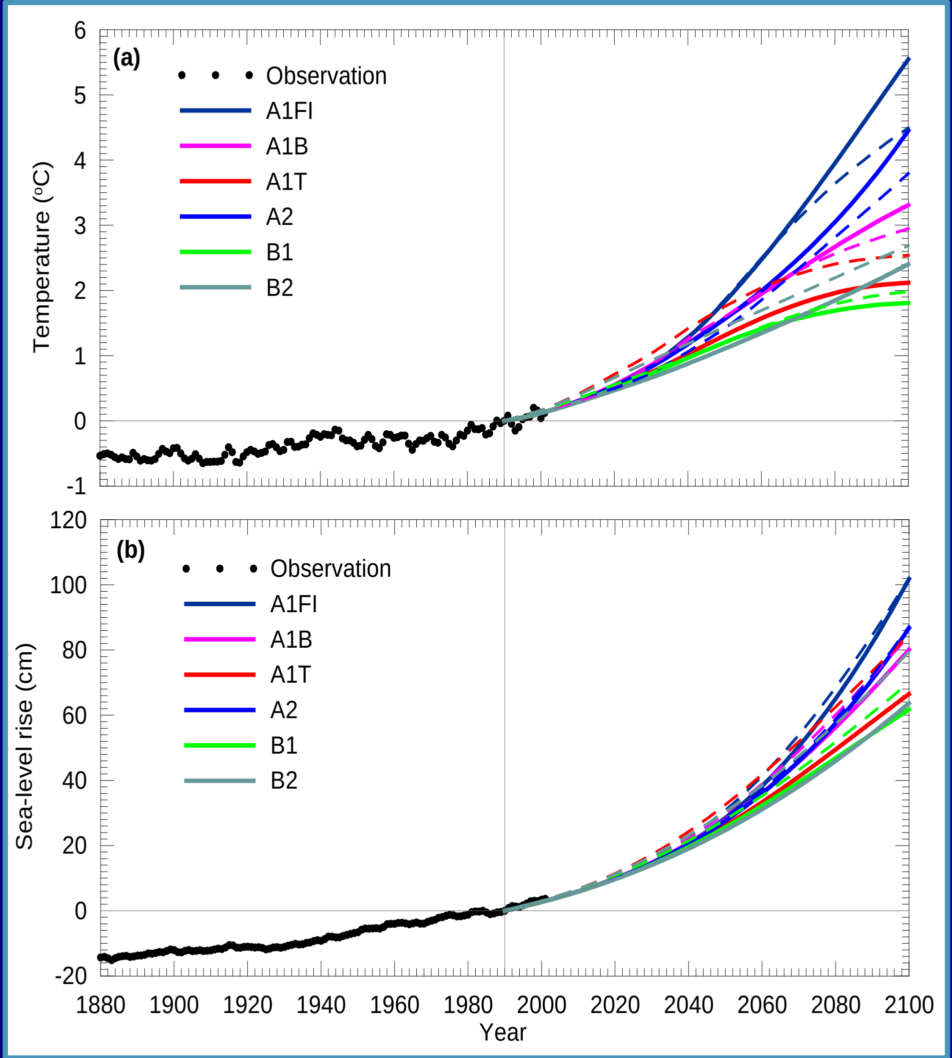
IPCC Global CO_2 Emission Scenarios:



Global CO_2 Impact: Comparison of results

Solid lines are
Dyn. Sys. Model results.

Dashed lines are IPCC results.



Decision Making on Global CO_2 Emissions:

For example: If we want to restrict the global temperature and sea-level rise to zero-growth, the global CO_2 emission should be controlled with the relationship given by

$$u_{CO_2}(t) = 101.87T(t) + 0.22H(t) + 349.98$$

where

$u_{CO_2}(t)$ represents the amount of yearly CO_2 emission (ppm).

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